Limits of imaging-system simplification using cubic mask wavefront coding

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Increasing the space-bandwidth product of an imaging system will lead to a complex and expensive optical system. Techniques exist to simplify imaging systems. We propose the use of a hybrid imaging system using pupil phase modulation. Based on the reconstructed image's mean-squared error, we compute how this error is affected under various third-order aberrations. We determine the best cubic phase-mask parameter and study the impact of the orientation of the coma and astigmatism, as we have in a real optical system (from 0 to 2π). We then compute how the reconstructed image's quality varies by adding defocus-related aberrations (defocus and/or field curvature). Based on our analysis, we determine the limits of a hybrid imaging system using a cubic phase mask to develop simplified imaging systems. We conclude that the simplified lens design can be corrected if its aberrations are limited to 1 lambda of coma, astigmatism, and spherical aberrations and less than 1 lambda of field curvature or defocus.

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Imaging systems for civilian and military applications are developed with an ever-increasing effective sensor resolution. To optimize the space-bandwidth product (SBP) [1], the imaging objective must be diffraction limited. To increase the information, the solutions are to reduce the pixel dimension, increase the sensor size, or both. This translates into larger aberrations to maintain the optimum SBP because the system has an increased field of view (FOV) and/or a lower f-number. Consequently, the imaging lens needs more correcting optical surfaces to correct the additional aberrations. This is of particular concern when a large FOV is targeted. Finally, the optical lens assembly becomes complex, expensive, and difficult to miniaturize.

This problem has been addressed in past decades. A most recent paper [2] reports using a monocentric lens for compact imagers. Using this design, the authors targeted a 100 megapixel resolution over 160 deg. Curved detectors can also help to produce a simple imager [3] because the field curvature is naturally corrected. Other approaches involved multichannel systems inspired by insect eyes [4], multiscale lenses [5], and folded thin lenses [6]. Strategies have been studied regarding the limits of using a single optical element combined with postprocessing [7]. Cossairt in a recent paper showed a scaling law for computational imaging [8]. The hybrid imaging system is a computational imaging technique, which is based on the use of a pupil modulation and postprocessing to extend the depth of field [9]. The basic concept is to introduce a large amount of aberration in the pupil to make the point spread function (PSF) invariant to defocus. The corresponding modulation transfer function should then be higher than zero to allow reconstruction of the image by deconvolution. The technique proved to be useful to mitigate focus-related aberrations [10], optical system tolerances, and cost [11]. This approach is a good candidate for developing simplified imaging systems [12] in terms of lens composition. A simplified lens design will not be corrected for all the aberrations, but, using a pupil modulation and proper deconvolution, the image quality can be recovered. However, the quality of the reconstructed image will be affected by these residual aberrations. The effect of the aberration other than defocus (and the effect of multiple aberrations at the same time) on the restored image is, to our knowledge, still poorly documented. Some studies extend to the invariance to third-order aberration [13–15]. These studies showed how the different types of phase modulation are more or less robust to astigmatism and coma. Unfortunately, the studies have been done with fixed-aberration orientation and not for aberration combinations. Moreover, it did not provide a clear range of aberrations that can be mitigated by the hybrid imaging system (except for defocus).

We report in this Letter for the first time quantitative aberration limits that a hybrid optical system can exhibit. We discuss how the reconstructed image is affected according to the variation of the aberrations (both in strength and orientation) within the FOV as it can be defined in a realistic lens system. We use a well-known image-quality matrix based on the mean-squared error and compute how this quality factor is affected under various third-order aberrations and cubic phase-mask strength. From these results, we determine the best cubic phase-mask parameter to study the impact of the orientation of the coma and astigmatism, which varies across the FOV, as we will have in a real optical system (from 0 to 2π). Finally, we compute how the reconstructed image quality (RIQ) varies by adding defocus-related aberrations such as field curvature.

A way to evaluate the hybrid imaging system's performance is to compare the final restored image to the perfect original image using mean-squared error [15,16]. This comparison with the original image is better than comparing with the ideal noise-free diffraction-limited image since hybrid imaging can achieve super resolution. The RIQ factor [16] can be expressed by Eq. (1):

$$\mathrm{RIQ} = -10\log_{10}\langle|\mathrm{RI}_{ij} - O_{ij}|^2\rangle, \quad (1)$$

where RI is the reconstructed image, O the object, i and j the coordinates of the pixels, and $\langle\,\rangle$ denote the average values over all the pixels. For simplicity, the maximum pixel intensity in the object and image is normalized to 1, and the object used is the Lenna image in 512×512 pixels.

To compare the hybrid image performance, we use the RIQ of an image generated with $\lambda/4$ of defocus, which corresponds to a diffraction-limited image according to Rayleigh criteria. We also assume that the image sampling is larger than Nyquist to avoid aliasing. Using this parameter, Eq. (1) gives an RIQ of 20. This is chosen to be the minimum acceptable quality. This means that if the hybrid system gives an RIQ of 20, this corresponds to a diffraction-limited performance.

Numerical simulations of optical systems with varying degrees of aberrations were made using Matlab and analyzed based on Eq. (1). The image (I) produced by the hybrid optical system is generated according to Eq. (2). The image is reconstructed as defined by Eq. (3) using a Wiener filter g [15].

$$I = \mathrm{PSF} \otimes O + n, \quad (2)$$

$$\mathrm{RI} = g \otimes I. \quad (3)$$

PSF is the point-spread function of the hybrid optical system with a circular aperture, including optical aberrations and the pupil phase modulation. n is the image noise. Gaussian white noise is added to be more realistic, and the Wiener filter is calculated accordingly. The image reconstruction assumes that the image is corrected for any shift introduced by the pupil phase mask [16].

The pupil function (P) of the hybrid optical system is given by Eq. (4):

$$P(x,y) = \exp\left\{\begin{array}{l}\left[i\frac{2\pi}{\lambda}\Big[W_{40}(x^2+y^2)^2\right.\\ +W_{20}(x^2+y^2) + W_{22}(y^2)\\ \left.+W_{31}(x^2+y^2)y\Big] + i\alpha(x^3+y^3)\right]\end{array}\right\}, \quad (4)$$

where α is the mask parameter in radiant, x and y the normalized pupil coordinates (x and y vary from -1 to 1), and W is the amplitude of the aberration in waves. The aberrations used in this Letter are the basic third-order aberrations: defocus (W_{20}), coma (W_{31}), astigmatism (W_{22}), and spherical aberration (W_{40}). The phase mask used is the cubic phase mask.

The z is calculated by the autocorrelation function of the pupil function $P(x,y)$ as defined in Eq. (5) where TF is the Fourier transform and P^* is the conjugate:

$$\mathrm{PSF} = \mathrm{TF}^{-1}[P(x,y) \otimes P^*(-x,-y)]. \quad (5)$$

For the reconstruction, the PSF used in the Wiener filter is calculated using Eqs. (4) and (5) where all the aberration coefficients W are equal to zero.

We first analyzed the impact of the cubic phase-mask parameter on the RIQ in the presence of third-order aberrations. Figure 1 gives the RIQ as a function of the level of aberration in waves and the phase-mask parameter α. As expected, in accordance with the literature, with higher defocus, a larger α parameter is required to obtain a better image quality. Note that when considering only defocus, some authors use a filter based on several defocus positions [16]. This technique could be interesting, but it is not used in our analysis. The position at $\alpha = 0$ represents a hybrid system without a pupil phase mask and reconstructed with a uniform aperture PSF.

It is interesting to note that, according to our simulation, the optimal phase-mask parameters are the same as those reported in the literature. For example, we have an optimal parameter α of about 18 for a defocus of 5λ. This value corresponds exactly to the 2.87λ reported by Vettenburg *et al.* [15] using the $2\pi/\lambda$ factor (different definition).

It can be appreciated from Fig. 1 that the optimal values of α are not the same for each aberration. For increasing values of α, the RIQ is less sensitive to defocus, but it is not the case for the other aberrations. This observation is, however, in contradiction with the results from Lei *et al.* [14]. For large values of α, the dependence on astigmatism, coma, and spherical aberration cannot be ignored for aberrations greater than 0.5λ. Using Fig. 1, we can determine that we have a zone with an RIQ higher than 20 for a phase-mask parameter interval between 4 and 6 and for less than 2λ aberration. For higher α values, higher defocus and astigmatism can be mitigated, but for spherical and coma, the aberration

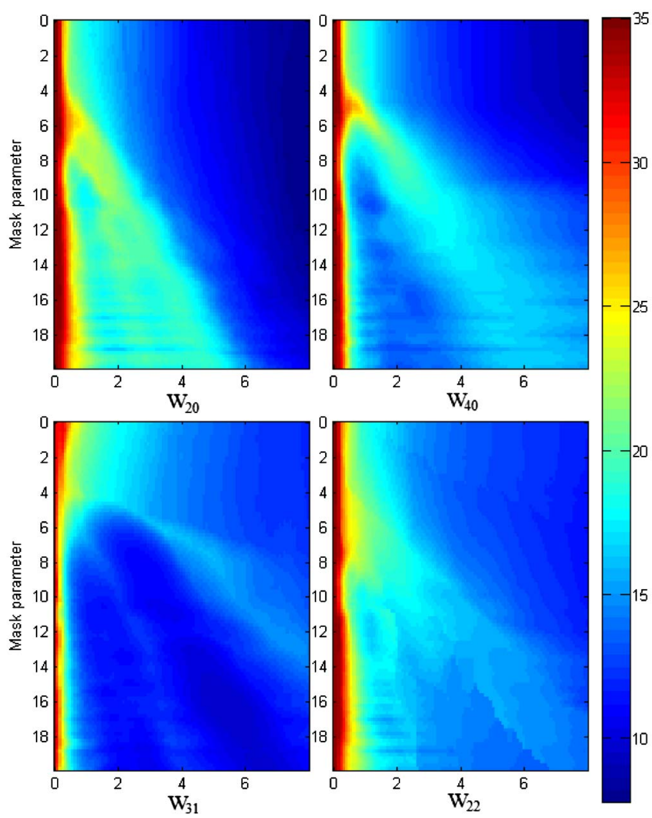

Fig. 1. RIQ as a function of aberration and cubic phase-mask parameter. No noise is added.

must be lower to be compensated. Coma seems to be the limiting aberration.

Moreover, coma and astigmatism aberrations do not have a circular symmetry. As off-axis aberrations, the orientation of these aberrations depends on the tangential and the sagittal optical planes. The aberration is oriented according to the tangential plane defined by the image height to the optical axis (radial direction center to the edge of the image). Consequently, the orientation of the coma and astigmatism will rotate within the FOV from 0 to 2π (azimuthal angle in the image plane). In Fig. 1, the RIQ is computed for the worst coma and the astigmatism orientation. Figure 2 shows a deeper analysis of the effect of PSF orientation on RIQ. We can appreciate on the figure the orientation of the wavefront coding PSF (upper left corner) and the relative orientation of the aberration according to the rotation angle. As we can see, the RIQ varies with the orientation of the aberrations. This means that if a system using wavefront coding with a cubic phase mask is affected by astigmatism and/or coma, then the image is going to present non-rotationally symmetric variations of the image quality. Minimizing such effect will be required. It can be seen that for larger aberrations (bottom graphics), even if we obtain an acceptable RIQ at an angle of zero, the variation over the FOV can be catastrophic in terms of image quality.

Simulations were also performed with various amounts of white noise. The noise type is not as important as the level [16]. Noise is amplified in the deconvolution process. Simulations have shown that, for moderate amounts of noise [signal-to-noise ratio (SNR) over 30 dB, $10\log_{10}(S/N)$], the best α parameter is unaffected and the RIQ is slightly reduced. For an SNR lower or equal to 30 the maximum α parameter that can be used decreases as the noise increases. This will decrease the maximum aberration that can be compensated.

The effect of multiple aberrations on a system with a phase mask is difficult to study because of the large quantity of calculations that need to be made. In the case of

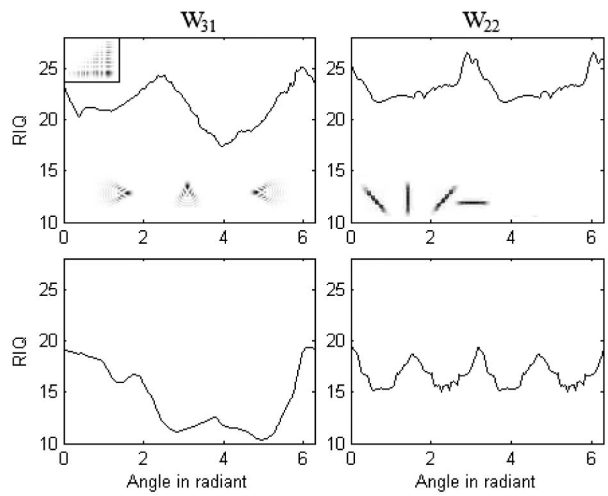

Fig. 2. RIQ as a function of the relative angle of the PSF for astigmatism and coma. The first row presents data for a phase-mask parameter $\alpha = 5$ and 1λ of aberration. In the second row, $\alpha = 15$ and the aberration is 3λ.

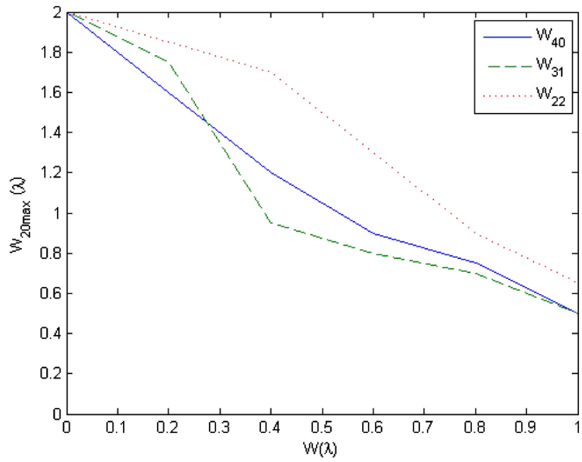

Fig. 3. Maximum compensated defocus as a function of the aberration coefficient for spherical, coma, and astigmatism.

simplified hybrid imaging systems, the lens construction will be simplified (less lenses), and the lens will have more aberrations. For the purpose of this Letter, the simulations were made to obtain the RIQ as a function of the defocus, α parameter, and the other aberrations (coma, spherical, or astigmatism). In Fig. 3, the maximum defocus that can be compensated (to maintain a RIQ larger than 20) is shown with the aberration coefficient for the spherical, coma, and astigmatism terms. It can be seen that the maximum defocus decreases monotonically with the aberrations for all three selected aberrations, and the astigmatism provides a slightly higher maximum defocus. This modest higher tolerance to astigmatism can be explained by the nature of astigmatism, which is similar to a defocus.

In Fig. 4, the optimum α parameter that produces the maximum defocus compensation according to Fig. 3 is shown with the aberration coefficient for the spherical, coma, and astigmatism terms. As the aberration is higher, the optimum phase-mask parameter decreases in all cases. The coma is interesting because, for higher aberration, the phase-mask parameter is closer to zero, which means that having or not having a phase mask will not improve the performance of the reconstructed image.

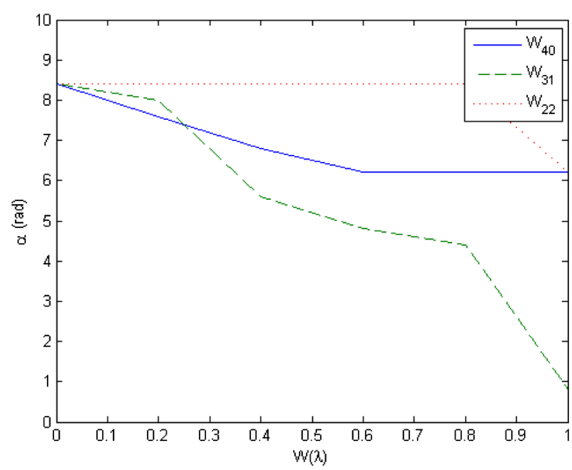

Fig. 4. Optimal cubic phase-mask parameter as a function of the aberration coefficient for spherical, coma, and astigmatism.

We have also performed simulations using 34 dB of noise, and the results were mostly the same.

Wavefront coding using a cubic phase mask offers improved capabilities to mitigate defocus aberration by increasing the depth of field of a lens. It can also be used to compensate for other types of aberrations, even some off-axis aberrations. However, if we want to use this technique to reduce the complexity of an optical system, very limited data is available. We have analyzed in this Letter the impact of several aberrations on the RIQ.

Based on a standard image mean-squared error called RIQ, we determined that the variation of the RIQ was very different from various aberrations. The coma appeared to be the most limiting factor. Using other types of error function and the deconvolution method may lead to different results, but we think it will not change our main conclusion. We showed that this limitation was due to the variation of the orientation of the coma within the FOV. Most of the previous studies have been done with fixed coma orientation, which is why we obtained a different behavior. We have clearly shown that orientation was a key factor during the design; a good RIQ at a particular orientation can lead to a drastic reduction of the quality factor at other positions in the FOV. Finally, we showed that a cubic phase mask used to compensate for coma, astigmatism, and spherical aberrations had a limited potential to correct for field curvature and/or defocus.

The conclusion is that the use of a cubic phase mask in a hybrid system to reduce the complexity of the optical imager is limited. The simplified optical system must have residual aberrations of a maximum of 1 lambda of coma, astigmatism, and spherical aberrations and less than 1λ of field curvature. Moreover, the design of such a system must be done using various fields of view in the entire image because of the orientation of the off-axis aberrations. This makes the use of such hybrid techniques more complex. Additional studies are under development regarding other phase-mask types, noise levels, sensitivity to higher-order aberrations, and/or fabrication errors. Using a combined aberration to develop the reconstructed PSF is a technique that can be interesting to study in order to improve performances.

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