Graded-index planar waveguide solar concentrator

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Planar waveguides are useful to transport, concentrate and distribute light uniformly over large dimensions. Their capacity to collect and gather light efficiently over a large distance is interesting for many applications, like backlighting and solar concentration. For these reasons, the possibility of making them even more efficient could be of considerable interest for the community. The observation of the ray path inside a graded-index (GRIN) fiber inspired the development of a similar technology inside planar waveguides. In this Letter, we show that it has the potential to dramatically increase the efficiency of planar waveguide-based solar concentrators or backlighting using GRIN planar waveguides.

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Planar waveguides, as a backlighting device, have been in use for a long time [1]. This has inspired the development of solar concentrators based on very similar designs [2–4]. For both uses, the optical efficiency of the system is of great importance. In the case of backlighting systems, you want to precisely control the extraction of light from the waveguide to get uniform illumination and, in the case of solar concentrators, you want to reduce the propagation losses as much as possible.

In the latter case, the light coming from the sun is concentrated by an array of lenses and coupled into the waveguide using a coupling structure that is, habitually, an array of reflecting prisms. The light deviated by the prisms propagates by total internal reflection (TIR) in the multimode waveguide to reach the solar cell at the edge. Figure 1(a) shows the planar waveguide system, where the cone of light having an angle θ is redirected by the coupling structure into the waveguide. Ideally, the rays coupled into the waveguide will propagate until they reach the solar cell on the side. However, due to the presence of many coupling structures, undesired reflections of light propagating within the waveguide may happen, causing light previously undergoing TIR to be redirected at an angle that allows it to escape from the waveguide. Consequently, the losses for this kind of solar concentrator are, to some extent, proportional to the number of interactions of the light rays with other coupling structures within the waveguide. These losses will limit the length of the waveguide as well as the concentration factor. This is even more important if the waveguide uses cylindrical lenses [4]. Planar waveguide concentrators using cylindrical lenses as focusing elements are of interest because they can be used with only one axis of sun tracking, which results in cost saving [4].

We propose a new method to eliminate the light losses produced by the multiple interactions between the light propagating into the waveguide and the coupling structure. This promising method was inspired by ray propagation inside a graded-index (GRIN) optical fiber [5]. The benefit of GRIN optics has recently become of interest because of advanced material manufacturing techniques, particularly in polymers. The use of a nanolayered polymer, a new class of optical material with a variable index profile and high index change, has been demonstrated [6,7]. In a GRIN medium, the optical rays follow curved trajectories instead of straight lines. The trajectory of the rays is calculated by using Fermat's principle and according to the refractive index profile in the medium. The index profile can be chosen in such a way that the rays oscillate around the bottom of the slab with a period F (period of the gradient). In our case, the rays coming from the coupling structure will be re-imaged at a period F all along the waveguide, as shown in Fig. 1(b). Using a period F, which is different from the period of the coupling structure, we can avoid the light hitting a second coupling structure. Thus, there is no loss of light coming from the interaction with a coupling structure during propagation in the waveguide. Consequently, the length of the waveguide, as well as the concentration factor, will be significantly increased.

In this Letter, we present an analytical model to design the ideal GRIN planar waveguide concentrator. We also discuss the physical limits of these various parameters. The model is used to determine the maximum concentration factor, according to the lens f number ($F/\#$). Finally, we compare our GRIN planar waveguide concentrator with a step index planar waveguide concentrator.

The analytical model is built from Fig. 1(b), representing an array of cylindrical lenses having a focal length f (in the air) with a lateral dimension D on top of a GRIN planar waveguide with a maximum index of refraction n_0. The index of refraction in the waveguide varies from the top to the bottom with the higher index at the bottom. The concentration factor of the lens C_{lens} is given by Eq. (1) using the angular extension of the sun of 0.52° (2φ):

$$C_{\text{lens}} = \frac{D}{d} = \frac{D}{2fn_0\tan(\varphi)} \approx \frac{110}{F/\#}. \quad (1)$$

For the analysis, we have neglected the variation of the index of refraction to estimate the $F/\#$. A small prism structure, having the size of the sun image d produced by the lens, is located at the base of the waveguide. The sun is imaged on a metalized prism structure and

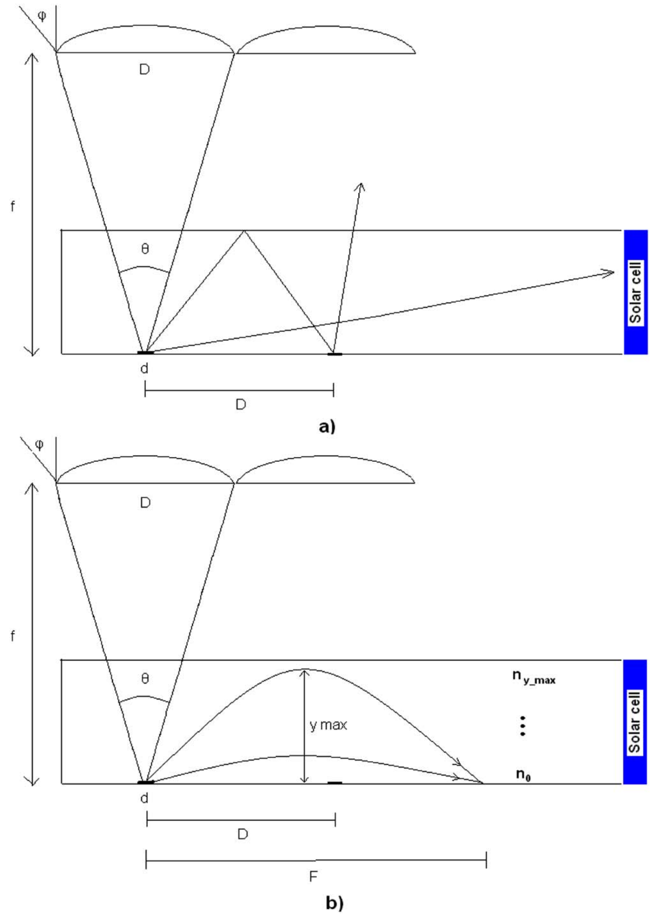

Fig. 1. (a) Light losses in a homogeneous planar waveguide. (b) Optical system developed in this Letter. This is a graded-index (GRIN) planar waveguide of thickness equal to the maximum height reached by the rays of light $y_{\max}$ and with an array of cylindrical lenses. Each lens has a width D and a focal length f. It produces a spot size d and the angular extent of the rays coupled in the waveguide is θ. The spot is re-imaged at a distance F, which is the period of the GRIN. The angular acceptance of the optical system is φ. Finally, the index at the base of the waveguide (opposed to the lens array) is n_0.

deviates into the waveguide. The light will then travel into the GRIN waveguide by oscillating with a spatial period between the bottom and a maximum height ($y_{\max}$) until it reaches the exit aperture (side). At the bottom, the rays are reflected by TIR. Since the number of periods F of the light in the waveguide is slightly different from the distance D between two coupling prisms, rays will travel for a certain waveguide length L without any losses. The parameters of the system can be chosen to maximize the propagation length. The optical efficiency, as well as the concentration factor of the system, will be maximized.

The period of the gradient F is given by Eq. (2):

$$F = MD \pm kd. \tag{2}$$

M is an integer larger than zero and k is a number between 1 and D/d. It represents the difference between the period F and the distance between two coupling prisms. As shown in Eq. (2), with $1 < k \le D/d$, F is then slightly different than D (even for $M = 1$). To be realistic, the parameter k must be larger than 1 because the structure that deviates the light will have a certain dimension that will be larger than d. Consequently, to avoid losses and vignetting, the size of the coupling structure must be used in Eq. (2), and it is represented by kd.

The period of the gradient F is used to determine the number of lenses and coupling prisms of the concentrator. Equation (3) gives the number of lenses N in the array, as well as the number of coupling prisms at the bottom of the GRIN planar waveguide:

$$N = \frac{F}{kd} = \frac{(MD \pm kd)}{kd} = \frac{MC_{\text{lens}}}{k} \pm 1. \tag{3}$$

It must be noted that N is an integer (lowest value). The length L of the GRIN planar waveguide is determined by the length of the lens array. This is similar to the system proposed by Karp *et al.* [3]. Thus, the length is the number of lenses N multiplied by the lens dimension D, according to Eq. (4):

$$L = ND = D\left(\frac{MC_{\text{lens}}}{k} \pm 1\right). \tag{4}$$

The maximum concentration factor C can then be calculated simply by dividing the length of the waveguide by the maximum thickness $y_{\max}$. This leads us to the expression in Eq. (5). In this equation, R is added to represent losses associated with the reflection at the coupling prism:

$$C = R\frac{D\left(\frac{MC_{\text{lens}}}{k} \pm 1\right)}{y_{\max}}. \tag{5}$$

The dimension $y_{\max}$ relates to the index profile of the GRIN planar waveguide. According to the Fermat principle developed for GRIN optical fiber, the ideal lateral index gradient is a hyperbolic secant index profile [8]. For the purpose of this analytical development, we approximated the index profile by Eq. (6). It can be shown that the Taylor expansions of both ideal and approximated profiles are similar:

$$n(y) = n_0(1 - A^2y^2)^{1/2}. \tag{6}$$

The maximum index of refraction is given by n_0 and A is the index profile parameter. According to Eq. (6), $y_{\max}$ can be determined by solving the equation at $y = y_{\max}$ ($n_{y\,\max}$):

$$y_{\max} = \frac{\sqrt{n_0^2 - n_{y\,\max}^2}}{A \cdot n_0}. \tag{7}$$

Equation (5) can then be expressed by Eq. (8):

$$C = R\frac{D\left(\frac{MC_{\text{lens}}}{k} \pm 1\right)}{\frac{\sqrt{n_0^2 - n_{y\,\max}^2}}{n_0 A}}. \tag{8}$$

Since we have light coupling on the back surface, there is a relation between the maximum height of the rays propagating in the waveguide, the angular acceptance of the waveguide, and the index profile parameter A. In Eq. (8), the expression $(n_0^2 - n_{y\,\max}^2)^{1/2}$ is the numerical aperture (NA) of the GRIN waveguide side coupling. The relation between the propagation angle within the waveguide and the NA is the index of refraction n_0. Consequently, the minimum $F/\#$ of the lens or the maximum NA of the lens that can be guided by the GRIN planar waveguide is given by Eq. (9) (considering the small angle approximation):

$$F/\#_{\min} \approx \frac{n_0}{2\sqrt{n_0^2 - n_{y\,\max}^2}} = \frac{1}{2Ay_{\max}}. \tag{9}$$

Equation (9) shows that, for a given lens $F/\#$, it is possible to find a combination between A, $y_{\max}$, and n_0 to satisfy this condition. Consequently, the design of the GRIN waveguide is limited by the manufacturing constraints. The maximum and the minimum index of refraction are limited to some extent. If the variation is limited to 10%, the minimum $F/\#$ for a waveguide with $n_0 = 1.6$ is about 1.15. This number is far from the small angle approximation, but it shows that only 10% index variation is sufficient for most of the applications where the $F/\#$ is generally larger than 1.

Equations (1)–(9) give the basis of the GRIN planar waveguide solar concentrator. To illustrate the benefit of the proposed approach, we will show a comparison between a planar step index waveguide concentrator [3] and the new GRIN planar waveguide concentrator.

Figure 2 shows the concentration factor for both step and GRIN index planar waveguide concentrators. The concentration factor for the GRIN planar waveguide is calculated using Eq. (5). The equation is displayed for M equals 1, a k factor of 2, a lens diameter D of 2 mm, $y_{\max}$ of 1 mm, and $n_0 = 1.55$. The chosen value of R is 0.92.

Equation (5) shows that the $F/\#$ of the optical system is a key parameter for the GRIN planar waveguide solar concentrator. Modifying the $F/\#$ will directly affect C_{lens} and the index constant A so that light will propagate without losses. To maximize the system efficiency, we want to fill the entire waveguide with light over the full thickness, since it maximizes the number of modes coupled. Therefore, we use the optics with the lowest possible effective $F/\#$. As can be seen in Fig. 2, this leads to an increase in the concentration factor of the system compared with a step index planar waveguide. The objective when designing a solar concentrator is to reach the highest concentration factor possible. Thus, for the same $F/\#$, the GRIN waveguide will reach a higher concentration factor than the homogeneous one, which is advantageous.

The concentration factor for the step index waveguide is derived from the work by Karp *et al.* [3]. The homogeneous waveguide, with coupling prism structures on the back surface, is the reference point for planar concentrators. In such cases, the light propagates in the waveguide only by TIR, since it uses a homogeneous material. The analytical model to a planar concentrator

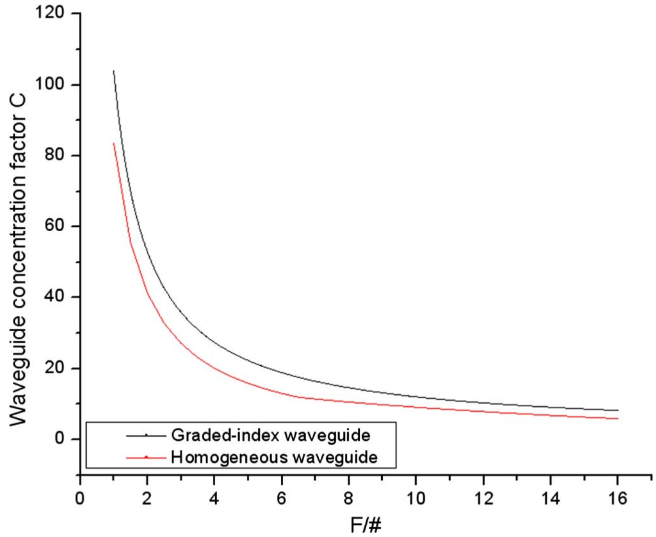

Fig. 2. Concentration factor for various $F/\#$ of the cylindrical lens for the GRIN and the homogeneous planar solar concentrator. The chosen value for the parameter k is 2. The diameter of the lens D is 2 mm. The thickness of the waveguide $y_{\max}$ is 1 mm. The index at the base of the GRIN waveguide n_0 is 1.55.

with cylindrical lenses has been studied in previous work [4]. The main equations used to calculate the optical efficiency are presented in Eqs. (10)–(12). These are based on the probability of a ray being stopped by a prism during propagation in the waveguide. However, since prism width does not necessarily fit the spot size produced by the lens, C_{lens} is divided by the parameter k in the calculation of the probability. This is the same parameter k that was used for the GRIN waveguide. We recall that the total optical efficiency is calculated by summing individual efficiency for each lens ($2r$) at each position (P) in the waveguide, and for all angles coupled (ϕ). R is the reflection coefficient of the coupling prisms and α is the attenuation coefficient of the material:

$$\eta_{\text{decouple}}(P,\phi) = \left(1 - \frac{k}{C_{\text{lens}}}\right)^{\frac{P\tan\phi}{2y_{\max}}}, \tag{10}$$

$$\eta_{\text{position}}(P,\phi) = R \cdot \eta_{\text{decouple}} \exp\left(\frac{-\alpha P}{\cos\phi}\right), \tag{11}$$

$$\eta_{\text{total}} = \frac{\sum_P \int_0^{\phi_{\max}} \eta_{\text{position}}(P,\phi)}{\frac{(L-r)}{2r}}. \tag{12}$$

These equations are used to calculate the optical efficiency of propagation. The concentration factor is obtained by multiplying the optical efficiency by the geometric concentration factor. The geometric concentrator is simply the length of the waveguide divided by the waveguide thickness. Then, the concentration factor is calculated for each value of the $F/\#$. The $F/\#$ is equal to $1/(2n_o\,\tan(\phi_{\max}/2))$. The results are presented in Fig. 2 in the same format as for the GRIN waveguide. The effect of varying the $F/\#$ is the same for both systems. There is a continuous decrease in the maximum concentration

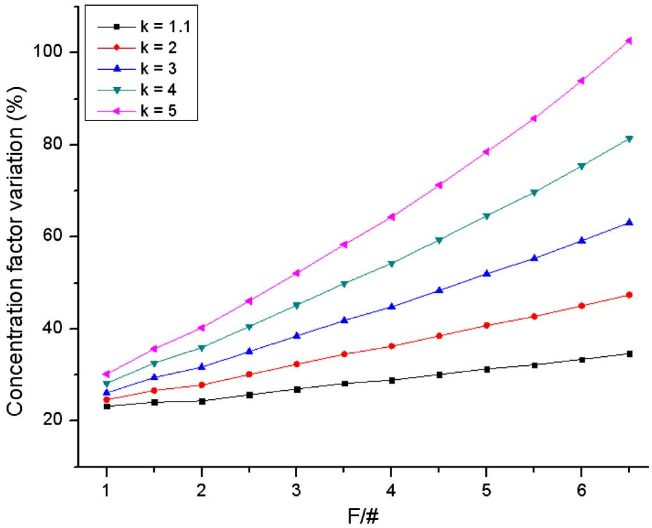

Fig. 3. Variation of the concentration factor associated with the use of a GRIN waveguide instead of a homogeneous waveguide. The homogeneous waveguide in the ratio is calculated with C_{lens}/k. Only optical systems with $F/\#$ that can be used in a compact concentrator with a good concentration factor are presented.

factor. This diminution of the maximum concentration factor attainable with an increase of the $F/\#$ has previously been described by Karp [9].

It appears that the theoretical performance of a concentrator using a GRIN waveguide with cylindrical lenses is significantly increased compared with the same concentrator using a homogeneous material. The variation of the concentration factor between both cases is presented in Fig. 3 over a range of $F/1$ to $F/6.5$ for different values of k. Concentration factor variation is expressed as the percentage of the variation between both curves presented in Fig. 2. These $F/\#$ correspond to the most promising range for a compact system, with a good concentration factor. For an $F/1$ lens, the concentration factor increases by about the same number for all values of parameter k. It is an increase of 23.2% for $k = 1.1$ and 30.2% for $k = 5$. It appears that there is a linear relationship between the concentration factor variation of the two different waveguide types and the $F/\#$. For different values of k, the slope of the variation is modified. For a more practical system with $F/6.5$, the difference between the GRIN and the homogeneous waveguide is even more impressive for all values of k. In this situation, the increase for $k = 1.1$ will be 34.7%, which is modestly better than for $F/1$. However, with $k = 5$, the concentration factor increase at $F/6.5$ is 102.7%. This is 3.4× better than with a $F/1$ lens. These results show that, in all situations, the GRIN system will reach a higher concentration factor. This increase in the concentration factor is also achieved without losses in the optical efficiency, which can be a critical factor when the size of the system is a constraint.

In conclusion, we have proposed in this Letter a new method to design a more efficient planar waveguide solar concentrator. This method was inspired by the behavior of light into a GRIN fiber. By using a GRIN parabolic profile within the planar waveguide, it is possible to eliminate the light loss produced by the multiple interactions with the coupling structure at the bottom of the waveguide. The benefit of polymer GRIN optics has recently generated interest because of advanced material manufacturing techniques. In a GRIN medium, the optical rays follow curved trajectories instead of straight lines. The trajectory of the light rays is calculated by using Fermat's principle according to the refractive index profile in the medium. The index profile can be chosen in such a way that the rays oscillate around the bottom of the slab with a period F. When using a period F that is different from the period of the coupling structures, we can avoid the light hitting a second prism. Consequently, the length of the waveguide as well as the concentration factor will be significantly increased.

This Letter described the basic equations required to design and optimize the GRIN planar waveguide. We have shown that, for a given lens $F/\#$, it is possible to find a combination of waveguide index profile, length, and width to maximize the concentration factor. We compared the concentration factor between a GRIN and a step index planar waveguide solar concentrator as a function of the lens $F/\#$. The theoretical results show that the GRIN planar waveguide reaches higher values if we carefully choose the parameter k. We showed a large increase of the concentration factor for $1.1 \leq k \leq 5$.

More work will need to be done to look for the best configuration. We will have to find a balance between optical efficiency, concentration factor, and waveguide thickness in real application cases. Experimental validation will be realized to address fabrication cost and the actual performance of the system. The same reasoning applies to backlighting illumination systems, so this will be another design to develop based on this analytical theory.

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